CS 599: Rounding Techniques in Approximation Algorithms

Homework 4: Iterative Randomized Rounding and SDPs

Fall 2024

Do at least two of the following three problems.

1 Problem 1: Beck-Fiala

Use the iterative randomized rounding framework to prove a bound of $O(\sqrt{t \log n})$ for Beck-Fiala, where n is the number of edges and the hypergraph has maximum degree t.

2 Problem 2: Matroid Intersection with Concentration

Let $M_1 = (\mathcal{I}_1, E)$ and $M_2 = (\mathcal{I}_2, E)$ be two matroids over the same ground set E. Let x be a point in the matroid intersection polytope $P_{M_1 \cap M_2}$. Give a polynomial algorithm that given x produces a set $I \subseteq E$ such that:

1. $c(I) \leq 2c(x)$,
2. I contains a basis of M_1 and M_2,
3. For any set of elements $F \subseteq E$, $\mathbb{P}\left[|F \cap I| \geq 2(1+\epsilon) \cdot x(F)\right] \leq e^{-\Omega(\epsilon^2 \cdot x(F))}$.

Hint 1: Use the iterative randomized relaxation framework with a slight twist.

Hint 2: The set of constraints in $P_M^{\uparrow}$ can be uncrossed to form a chain for any matroid similar to the constraints for P_M. You may use this as a given if you'd like.

3 Problem 3: SDPs

Read Section 13.2 of the Williamson-Shmoys book and show that the algorithm there can be used to color a 3-colorable graph with $\tilde{O}(n^{1/4})$ colors (where $\tilde{O}$ hides polylogarithmic factors), i.e. do Exercise 13.1 in the Williamson-Shmoys book.